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ACCURACY OF ASYMPTOTIC SOLUTIONS FOR NONUNIFORM SUPERSONIC FLOW PAST A BLUNT BODY

N. N. Pilyugin, R. F. Talipov, and S. V. Utyuzhnikov

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The study of flow field and heat exchange about blunt bodies when the oncoming supersonic stream is substantially nonuniform has been of much practical interest lately. In [1] we considered the results of experimental and theoretical study of the resistance, heat exchange, and the gasdynamic picture of the flow past a pair of bodies, one of which is behind the other in a supersonic stream. The experimental results in [1] were obtained for a relatively small separation between the bodies (no more than 20 calibers). The study is carried out by theoretical methods when the bodies are separated by a large distance (several hundred calibers) [2-6]. In [2, 3] we obtained asymptotic solutions for the problem of the flow of a nonuniform wake-type stream past a blunt body for moderate ($Re_\infty < 10^3$) and high ($Re_\infty > 10^5$) Reynolds numbers. In [1, 4-6] as a result of numerical solution of the equations of a thin viscous shock layer (on the assumption that the shock wave is equidistant from the surface of the body) we obtained relations for the heat exchange, friction, and the criterion of flow without separation as functions of the parameters of the problem.

For a uniform mainstream the method of a thin viscous shock layer (TVSL) gives results that are in satisfactory agreement with calculations with more exact methods [7, 8]. The applicability of the TVSL method [4-6] and asymptotic formulas [2, 3] to the case of a nonuniform supersonic stream of the far wake type flowing past a blunt body has not been examined sufficiently.

It is a particularly complicated matter to prove that the asymptotic expansions converge to an exact solution of the problem when there are several small parameters in which the expansions are made ($\varepsilon = \rho_\infty/\rho_s$, $Re_\infty^{-1/2}$, M_∞^{-2}, etc., where ρ_∞ and ρ_s are the density in the mainstream and behind the step, Re_∞ and M_∞ are the Reynolds and Mach numbers). The answer to these questions can be obtained either from systematic comparisons of the calculated gasdynamic parameters over a wide range with the results of specially designed aerodynamic experiments (which are often complex or impossible) or from a comparison with numerical solutions of more exact (unsimplified) equations of gasdynamics.

This study of a nonuniform supersonic flow past blunt bodies is based on the equations of a complete viscous shock layer (CVSL), which are solved numerically by using the effective method of global iterations [7-11]. The high accuracy and speed of this method as applied to CSVL equations have been confirmed by a comparison with experiment and the results of numerical solution of the Navier–Stokes equations by the method of fixing [7, 9, 10].

As shown here, in the case of low Reynolds numbers (Re_∞ = 50-100) the asymptotic formulas for a heat flux from [2] give practically the same results as does the numerical solution of TVSL equations. A comparison of the numerical solution of TVSL and CVSL equations showed that for a wake-type nonuniform mainstream the TVSL method leads to substantially underestimated (by up to 40%) values of the heat flux in the vicinity of the critical point. The critical values of the distance between two bodies (one of which is in the wake of the other), at which a separation zone arises on the frontal surface of the rear body, calculated by the TVSL method is approximately 1.5-2 times those obtained by using the CVSL equations.

1. Formulation of the Problem. The steady-state supersonic flow of a nonuniform stream of a viscous ideal gas (of the far wake type) past a smooth blunt body is considered. A system of CSVL equations in variables of the Dorodnitsyn type is given in [9, 10] and the boundary conditions are expounded in detail in [7]. We used the finite-difference method of solving equations of a higher order of accuracy, in much the same way as in [7-9].

After solving the difference equations, we calculate the distributions of the dimensionless surface heat flux q_w, the friction coefficient C_f, and the deflection δ of the shock wave as a function of the longitudinal coordinate x, the nonuniformity parameters a, b, c [7], the Reynolds number $Re_\infty = \rho_\infty V_\infty R_0/\mu_\infty$, and the Mach number M_∞.

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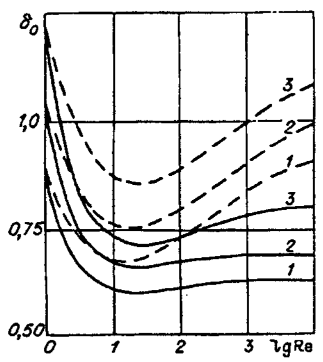

Fig. 1

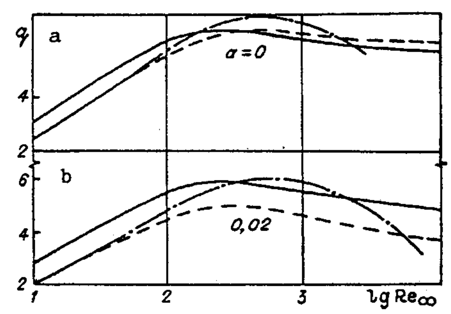

Fig. 2

2. Accuracy of the Asymptotic Solutions for Moderate Reynolds Numbers. In [2] we obtained an asymptotic solution in the vicinity of the critical line of the equations of the TVSL for $Re \leq 10^3$, $M_\infty \gg 1$, $\varepsilon Re = O(1)$. We ascertain the accuracy of the asymptotic formulas given in [2] by comparing them with the numerical solution of the TVSL and CVSL. Figure 1 shows the plot of the deflection of the shock wave $\delta_0 = y_{s0}/\varepsilon R_0$ [where $\varepsilon = (\gamma - 1/2\gamma)$, $Re = \rho_\infty V_\infty R_0/\mu_0$, $\mu_0 = \mu(T_0)$, T_0 is the stagnation temperature] on the Reynolds number for the Prandtl number $Pr = 0.5$, $H_w = 0.15$, $M_\infty \to \infty$, $b = 7.2$, $c = 3.0$, for $a = 0$, 0.02, and 0.04 (curves 1-3, respectively). The solid curves correspond to the TVSL numerical solution and the dashed curves, to calculation from the formula from [2]. We note that for large Re numbers the curves reach the asymptotes. The deflection of the shock wave increases as Re decreases.* An increase in the nonuniformity parameter a results in a considerably large shock-wave deflection for all Re. With increasing a and Re the agreement between the numerical and analytical solutions for the shock-wave distance becomes worse (the difference is more than 20% for $Re > 10^2$). Figure 2 shows the dependence of the heat flux $q = C_H\sqrt{Re_\infty}$ at the critical point on Re_∞, calculated from the CVSL model (solid lines), TVSL model (dashed lines), and the formula from [2] (dash-and-dot lines) in uniform (a) and nonuniform (b) streams. The other parameters were: $M_\infty = 20$, $T_w/T_0 = 0.1$, $Pr = 0.7$, $b = 7.2$, and $c = 3$. From Fig. 2 we see that the analytical solution and TVSL calculation are in good agreement in the uniform and nonuniform cases for $Re_\infty \leq 500$. The agreement of the TVSL and CVSL results becomes appreciably worse as a and Re_∞ increase. The results of analytical solution [2] agree to within 10% with calculation of the CVSL equations in the range $10^2 \leq Re_\infty \leq 3\cdot10^3$.

3. Criterion for Simulation under Nonuniform Flow. As shown in [3-7], when the model of a hypersonic (thin) viscous shock layer is used the effect of the nonuniformity of the oncoming wake-type stream manifests itself in the neighborhood of the critical line through the parameter (as $\varepsilon \to 0$)

$$\lambda = \frac{2ab(1 + c)}{1 - a},$$

which appears in the series expansion of the pressure gradient $\partial p/\partial x$ in powers of x. Calculations with the CSVL model show that λ cannot be used as a similarity criterion for the flow of a nonuniform wake-type stream past blunt bodies. For proof of this assertion we consider a comparison of the results.

Figure 3 shows the λ-dependence of the heat flux q_w at the critical point (made dimensionless in accordance with [5, 6])

$$\bar{q} = \frac{q_w}{B(1 - a)}, \quad B = 1 + c\left(1 - \frac{1}{(1 - a)^2}\right) \tag{3.1}$$

*The numerical and analytical results for $10 < Re < 50$ are preliminary and qualitative and need to be refined in the region of the transition to models of a rarefied gas.

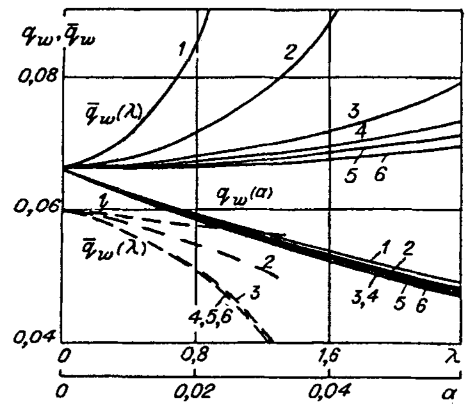

Fig. 3

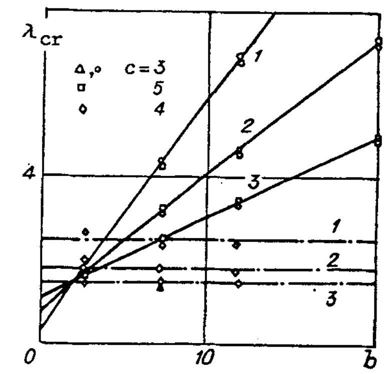

Fig. 4

for the following values of the determining parameters: $M_\infty = 20$, $Re_\infty = 10^4$, $T_w/T_0 = 0.1$. Curves 1-6 correspond to b = 1.7, 3.05, 5.4, 7.2, 9.6, and 12.8, while c has a constant value of 3. The solid lines correspond to the solution of the CVSL equation and the dashed lines, the TVSL equation. From Fig. 3 we see that as b decreases ($b \leq 3$) the curves for $\bar{q}_w$ separate, especially for more exact CVSL equations. The form of the dependence $\bar{q}(\lambda)$ differs for CVSL and TVSL. In both cases $q_w(a)$ decreases monotonically. The difference in the functions $\bar{q}(\lambda)$ is due, first, to the more rapid decrease in the heat flux q_w with growing nonuniformity (parameter a) in the case of TVSL, this being attributed to the assumption of an equidistant shape of the shock wave and the shape of the body and, second, it is due to the normalization of q_w by Eq. (3.1) to a complex B $(1 - a)$ that is variable in a. Moreover, the curves of the heat flux q_w (corresponding to the solution of the CVSL equation), plotted as a function of a for the same values of b, lie in a narrow pencil, as is seen from Fig. 3 (the solid curves 1-6, at the bottom).

A more rigorous calculation from the CVSL model demonstrated that the use of a has some advantages over the use of λ since the curves $q_w(a)$ are almost universal. In [5, 6] it is proposed that the criterion of transition to separating flow $a_{cr} = a_{cr}(b, c, Re_\infty, M_\infty, T_w)$, obtained in [1-4], be replaced by the criterion

$$\lambda_{cr} = \lambda_{cr}(Re_\infty, M_\infty, T_w).$$

The function $\lambda_{cr}(b)$ is plotted in Fig. 4 for $Re_\infty = 10^2, 10^3, 10^5$ (lines 1-3). The solid lines represent calculation from the CVSL model and the dashed lines, from the TVSL model [1, 4-6]. The effect of c on λ_{cr} is seen to be weak. The values of λ_{cr} obtained from the TVSL model are virtually independent of b and c for a fixed Re_∞. As $Re_\infty \to \infty$ the asymptotic value $\lambda_{cr} = 4/3$, obtained in [5, 6] gives good accuracy. Calculation from the CVSL model (solid lines) lead to a linear dependence of λ_{cr} on b for a fixed Re_∞. The plots shown in Fig. 4 support the conclusion of [5, 6] that the TVSL model leads to a separation criterion in the form λ_{cr} = const. Calculation from the more exact CVSL model, taking the transfer of perturbations upward along the stream, however, shows that the criterion λ_{cr} = const is unreliable.

As the separation criterion here we propose to use the formula

$$a_{cr} = \{1 + \varphi(b, c, Re_\infty)\}^{-1}, \quad \varphi = \frac{2b(1 + c)}{1.5 + (0.2 + 3.8\,Re_\infty^{-1/2})(b - 2)}, \tag{3.2}$$

obtained by processing the critical values of the nonuniformity parameters calculated from the CVSL model.

Using the criterion (3.2) and the formulas given in [1, 3], we can determine the critical value of the distance z_{cr} between two bodies at which a transition occurs to separation flow on the frontal surface of the rear body, in the supersonic far wake. Figure 5 shows the plots so obtained for z_{cr} as a function of the ratio of the diameter of the rear body to that of the front body, $d = d_2/d_1$, for fixed Reynolds and Mach numbers. The solid curves correspond to $M_\infty = 25$, the dashed lines to $M_\infty = 5$, lines 1-5 to the calculations for $Re_\infty = 10^2, 10^3$, and 10^5, and lines 4, 5 correspond to the critical values of z_{cr} obtained in [6] by using the criteria $\lambda_{cr} = 4/3$ and M_∞ = 25 and 5, respectively. These $z_{cr}(d)$ curves are also in good agreement with the results from [1, 3] as $\varepsilon \to 0$.

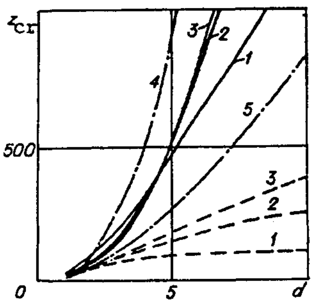

Fig. 5

From Fig. 5 we see that for small values of d (d $\leq$ 2) the TVSL and CVSL models give similar functions $z_{cr}(d)$, which also follows from the similarity of λ_{cr} for small *b* (see Fig. 4). For d > 2 the TVSL model gives values of z_{cr} that are higher by a factor of 1.5-2 than the CVSL results for $Re_\infty \to \infty$.

Calculation from the CVSL model shows that the values of z_{cr} increase with M_∞, Re_∞, and d. At moderately high Mach numbers (M_∞ = 5-8) the relative parameters of the bodies have little effect, especially for an intermediate Reynolds number ($Re_\infty \approx 10^2$-10^3). For $Re_\infty = 10^2$ and $M_\infty = 5$ the values of z_{cr} reach the asymptote for d $\geq$ 5. With decreasing Mach number the role of the relative dimensions falls off rapidly, the critical values of z_{cr} decreasing more rapidly than for moderate Mach numbers, and reach limiting values in the Reynolds number. The Mach and Reynolds numbers have only a weak effect on z_{cr} when the relative dimensions of the bodies are close (d = 1-2).

As a result of the calculations we have established that for each fixed Mach number there is a value d, for which the viscosity has no effect on z_{cr}, which is in accord with the asymptotic solutions [3]. The values of z_{cr} increase rapidly with increasing Re_∞ when $d > d_*$.

In summary, the examples of calculations given here show that the TVSL method becomes less applicable as the nonuniformity parameters increase. This can be attributed mainly to the assumption that the shock wave and the surface of the body are equidistant. The CVSL method has proven to work well for nonuniform flow past bodies and the proposed method of calculations required approximately two orders of magnitude fewer iterations than does the method of fixing for Navier–Stokes equations.

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